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Complementarity in variable strength quantum non-demolition measurements

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Abstract. Using a linear optic quantum gate we perform a variable strength quantum non-demolition measurement, to elucidate the role of which-path knowledge in a complementarity experiment. Specifically, we demonstrate that the entanglement created by the measurement interaction prevents an exhaustive description in terms of complementary wave-like and particle-like behaviour of a single photon in an interferometer.

Complementarity is one of the most distinct features of quantum mechanical systems [1]. A quantum object is unable to exhibit both perfect particle-like and wave-like behaviour in the same experiment. Put differently, any knowledge about the localization of the object prevents us from observing perfect quantum interference: this has been extensively tested in a variety of physical systems, including photons [2]–[4], atoms [5]–[7], molecules [8, 9] and electrons [10, 11].

A clear example is provided by the Mach–Zender interferometer. Here, light is split by a partially reflective mirror into two arms, then recombined on a second half-relfective mirror. The transmittivity of the first beam splitter gives us *a priori* knowledge about the likely path taken by the photon; this information prevents perfect interference between the two paths, except when the beam splitter is symmetric. In the general case, the which-path

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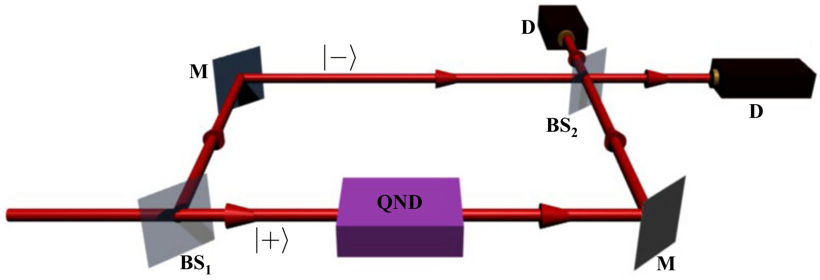

Figure 1. Schematic of the single-photon Mach–Zender interferometer with a QND device on one of the arms. M indicates fully reflecting mirrors and D single-photon detectors. The beam splitter BS_1 has variable transmittivity, while BS_2 is symmetric. The single photon is de-localized between the front rail $|+\rangle$ and the back rail $|-\rangle$. The presence of the photon on the $|+\rangle$ rail is revealed by the QND device with a given measurement strength. This determines the amount of knowledge that can be extracted *a posteriori*.

information is partial, resulting in a degradation of the visibility while still preserving some interference. A different way of extracting information is *a posteriori* by using a quantum non-demolition (QND) measurement on one of the arms [12, 13]; this is generally obtained by coupling the photon to some ancillary system, e.g. an atom. The stronger the coupling, the more information we can extract looking at the atom, resulting in a proportional degradation of the interference.

A precise relation between the which-path knowledge $\mathcal{K}$ and the interferometer fringe visibility $\mathcal{V}$ has been established by Englert [14], based on the previous results of Wootters and Zurek [15]; the visibility has an upper bound $\mathcal{V}^2 \leqslant 1 - \mathcal{K}^2$; this applies to both the *a priori* and *a posteriori* knowledge [16, 17]. In particular, pure states achieve the saturation of the bound when either *a priori* or *a posteriori* which-way knowledge is zero; in the general case, there are two Englert inequalities, each one holding independently of the other.

Intuitively then, it should be possible to combine the two Englert bounds for *a priori* and *a posteriori* knowledge to obtain a single complementarity relation between the *total* which-path knowledge and the visibility. This new inequality should also be saturated by pure states: it should be able to give us a strict bound to the visibility based on a complete information balance. In this paper, we demonstrate that this approach suffers several difficulties. We adapt the original proposal by Björk and Karlsson [18], to investigate the effect of both *a priori* and *a posteriori* information on the behaviour of a quantum system with an all optical setup. Our results show that a description in terms of single-particle duality cannot efficiently describe our experiment.

The conceptual layout of our experiment is shown in figure 1. There are two possible approaches to which-path knowledge. The *a priori* information $\mathcal{K}_s$ is set by the transmittivity T of the variable beam splitter BS_1. This produces a superposition of a photon in the front rail $|+\rangle$, and in the back rail $|-\rangle$: $|\sigma\rangle = \sqrt{T}|+\rangle + \sqrt{1-T}|-\rangle$. The knowledge $\mathcal{K}_s$ is related to the probabilities p_+ and p_- of finding the photon in the $|+\rangle$ or $|-\rangle$ arm respectively: $\mathcal{K}_s = |p_+ - p_-|$ [14, 16]. We can parameterize the transmittivity as $T = \sin^2\theta/2$, hence $\mathcal{K}_s = |\cos\theta|$. By Englert's formula, the resulting contrast at the outputs can be at most equal to $\mathcal{V}_s = |\sin\theta|$, occurring for an isolated system.

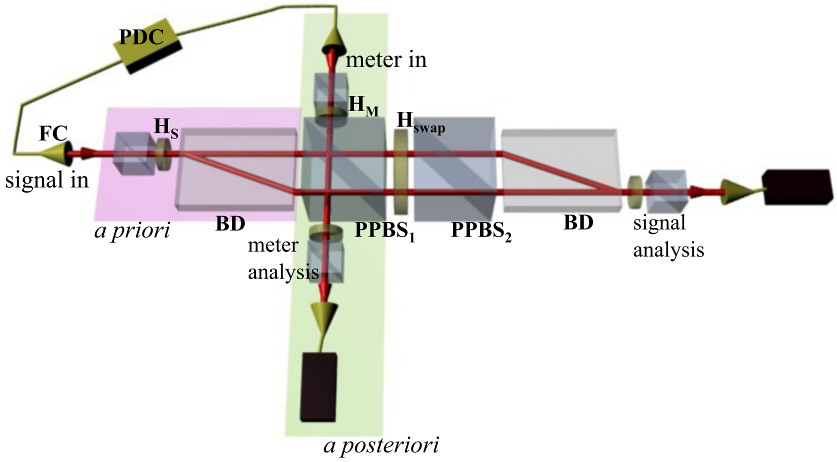

Figure 2. Scheme of the actual experimental setup. Photon pairs are produced by degenerate ($\lambda = 820$ nm) parametric downconversion (PDC) of 50 mW of frequency-doubled laser pulses ($\lambda_p = 410$ nm, $\Delta\tau = 100$ fs, rep. rate 76 MHz). Doubling and downconversion crystals are 3 mm-thick bismuth borate (BiBO) slabs cut for type-I phase matching. Single photons are spatially filtered by using single-mode fibre couplers (FC). Both the variable and the symmetric beam splitters are realized by the combination of a half-wave plate and a calcite beam displacer (BD). Half-wave plate H_S sets the polarization of the signal photon to $|\sigma\rangle = \cos(\theta/2)|H\rangle + \sin(\theta/2)|V\rangle$, which results in an effective transmittivity $T = \cos^2\theta/2$. Half-wave plate H_M sets the polarization of the signal photon to $|\mu\rangle = \gamma|D\rangle + \bar{\gamma}|A\rangle$, allowing to control the knowledge of the QND measurement. Non-classical interference between signal and meter occurs at the surface of $PPBS_1$. In order to recombine the two rails of the signal photon with a second identical BD, the polarization is rotated by an angle $\pi/2$ by means of the half-wave plate H_{swap}. The second $PPBS_2$ introduces polarization-dependent loss to ensure a correct balance of the polarization of the signal after the CZ gate. For the meter, loss is accounted for in the state preparation, as described in the text [24]. Polarization analysis is conducted on both outputs by a half-wave plate and a polarizer. Single-photon detection is performed by fibre-coupled avalanche photodiodes.

Previous investigations simulated the action of the QND by adopting maximally and non-maximally entangled photon states [19]: in that case, information on the signal was extracted by a measurement on the idler. Nevertheless, this procedure should rather be regarded as a remote state preparation [20], since the preparation and the measurement stage are unavoidably interwoven. Our implementation overcomes this difficulty by extracting *a posteriori* knowledge by inserting a photon number QND device in the front arm of the interferometer.

In the original proposal, this is realized by coupling the photon to an atom [18]. In our realization, we couple the occupation number of the rail to the polarization of an ancillary photon by a linear optical controlled-sign (CZ) gate inside a Jamin–Lebedeff interferometer, as shown in figure 2 [17, 21]. Such a configuration ensures long-term phase stability [22], and also allows one to measure the knowledge and the visibility by polarization analysis of the signal photon[5]. The gate is based on a single partially polarizing beam splitter (PPBS), whose transmittivities for the horizontal, H, and vertical, V, polarizations are ideally $\eta_{\mathrm{H}} = 1$ and $\eta_{\mathrm{V}} = 1/\sqrt{3}$ [23]. The $|+\rangle$ rail of the interferometer coincides with one input port of the gate. This corresponds to the upper rail in figure 2. In the other port, a single photon (the meter) is injected in a superposition of diagonal, D, and antidiagonal, A, polarization: $|\mu_{\mathrm{in}}\rangle = \gamma|D\rangle + \bar{\gamma}|A\rangle$, with $\frac{1}{\sqrt{2}} \leqslant \gamma \leqslant 1$, and $\gamma^2 + \bar{\gamma}^2 = 1$.

The presence of the signal in the $|+\rangle$ rail is detected since it causes quantum interference, and rotates the polarization of the meter to $|\mu_{\mathrm{out}}\rangle = \gamma|A\rangle + \bar{\gamma}|D\rangle$ [17]. When $\gamma = 1$, $|\mu_{\mathrm{in}}\rangle$ and $|\mu_{\mathrm{out}}\rangle$ are orthogonal, and can be discriminated perfectly by a measurement in the D–A basis; in the other cases, $\langle\mu_{\mathrm{out}}|\mu_{\mathrm{in}}\rangle \neq 0$, thus by a simple measurement we cannot conclude with certainty whether interference has occurred or not. The occupation number undergoes a weak measurement, which extracts only part of the information, while preserving some coherence in the measured photon [24]. The QND measurement can be characterized by its own knowledge[6], $\mathcal{K}_{\mathrm{m}} = 2\gamma^2 - 1$.

The CZ interaction U generally results in entanglement of the two photons; the joint state of the input and the meter after the interaction is $\rho_{\sigma\mu} = U|\sigma\rangle\langle\sigma| \otimes |\mu_{\mathrm{in}}\rangle\langle\mu_{\mathrm{in}}|U^{\dagger}$. Here and in the following, we use the label σ to identify the signal photon, and μ for the meter. The action of the gate also imposes losses on the H polarization of the input photons. In order to restore the correct balance at the output, one can either insert equal loss on the V polarization, or bias the input polarization to take it into account so to have a higher count rate. In our implementation, we adopted the first strategy for the signal photon by inserting a second PPBS, while we opted for prebiassing for the meter. This is a trade-off between having the highest possible count rate and achieving a satisfactory accuracy in the preparation of the signal.

The simplest complementarity analysis of our interferometer is concerned with how the presence of the QND device affects the which-way information and the visibility, without looking at the measurement results. For this purpose, we need to extend the definitions of such quantities in the state $\rho_{\sigma\mu}$. Following Bjork and Karlsson, we define the distinguishability $\mathcal{D}$:

$$\mathcal{D} = \left|\mathrm{tr}_{\mu}(\rho_{+} - \rho_{-})\right|, \tag{1}$$

where $\rho_{+} = {}_{\sigma}\langle+|\rho_{\sigma\mu}|+\rangle_{\sigma}$ and $\rho_{-} = {}_{\sigma}\langle-|\rho_{\sigma\mu}|-\rangle_{\sigma}$. Its meaning is the which-way knowledge $\mathcal{D} = |p'_{+} - p'_{-}|$, where $p'_{\pm}$ is the probability of finding the photon in the arm $|\pm\rangle$, after the measurement. In our system, the measurement leaves the initial probabilities unaltered, so that the distinguishability (1) coincides with the *a priori* knowledge: $\mathcal{D} = \mathcal{K}_{\mathrm{s}}$. The visibility $\mathcal{V}$ is given by

$$\begin{aligned} \mathcal{V} &= 2\left|{}_{\sigma}\langle+|(\mathrm{tr}_{\mu}\rho_{\sigma\mu})|-\rangle_{\sigma}\right| \\ &= \mathcal{V}_{\mathrm{s}}\sqrt{1 - \mathcal{K}_{\mathrm{m}}^2}. \end{aligned} \tag{2}$$

These two quantities need to satisfy Englert's bound $\mathcal{V}^2 + \mathcal{D}^2 \leqslant 1$, where the inequality holds also for an input pure state, differently from [16]. Figure 3 shows the measured complementarity relation over the range $0 \leqslant \theta \leqslant 2\pi$ for three different values of $\mathcal{K}_{\mathrm{m}}$. It shows good agreement

[5] Distinguishability is given by the contrast in the H–V basis, the visibility is given by the contrast in the D–A basis.

[6] In the experiment, the measurement strength can be measured from the coincidence probabilities: $\mathcal{K}_{\mathrm{m}} = P_{DD} + P_{AA} - P_{DA} - P_{AD}$, registered when the input of the Jamin–Lebedeff is diagonal [17]. See also [25].

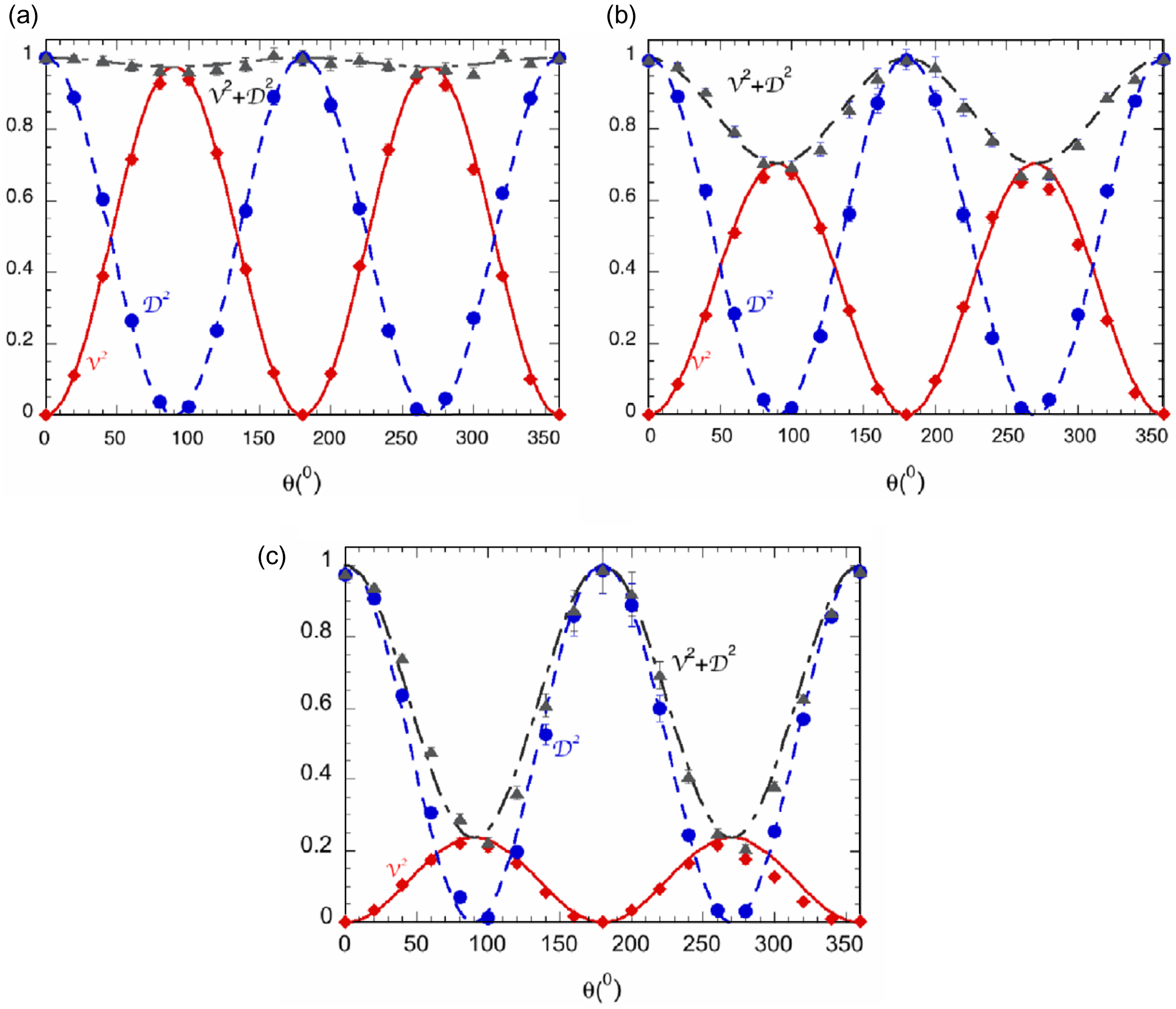

Figure 3. Test of Englert's complementarity relation: visibility $\mathcal{V}^2$ (red diamonds), distinguishability $\mathcal{D}^2$ (blue circles), and their sum $\mathcal{V}^2+\mathcal{D}^2$ (grey triangles) for three increasing values of the *a posteriori* knowledge: (a) $\mathcal{K}_m = 0.1598 \pm 0.0091$, (b) $\mathcal{K}_m = 0.5445 \pm 0.0083$ and (c) $\mathcal{K}_m = 0.8738 \pm 0.0058$. The solid lines are the theoretical predictions of equations (1) and (2). When not shown, error bars are smaller than the point size.

with the predictions. Despite the high level of purity of the initial state, it is clear that the inequality is not saturated: the QND interaction makes the photon an open system.

The quantities defined above are not suitable to find a saturated complementarity relation, since the distinguishability $\mathcal{D}$ is not affected by the *a posteriori* knowledge $\mathcal{K}_m$. We need to modify those definitions so as to include the information from the measurement into the visibility and/or distinguishability of the signal photon. As proposed in [18], we use the information we extract from the meter to sort the outcomes from the signal photon. Therefore, $\mathcal{D}$ and $\mathcal{V}$ are modified by splitting the measured ensemble according to the measurement result. The measured distinguishability $\mathcal{D}_M$ is defined as

$$\mathcal{D}_M = \left|{}_{\mu}\langle D|\,(\rho_+ - \rho_-)\,|D\rangle_{\mu}\right| + \left|{}_{\mu}\langle A|\,(\rho_+ - \rho_-)\,|A\rangle_{\mu}\right|, \tag{3}$$

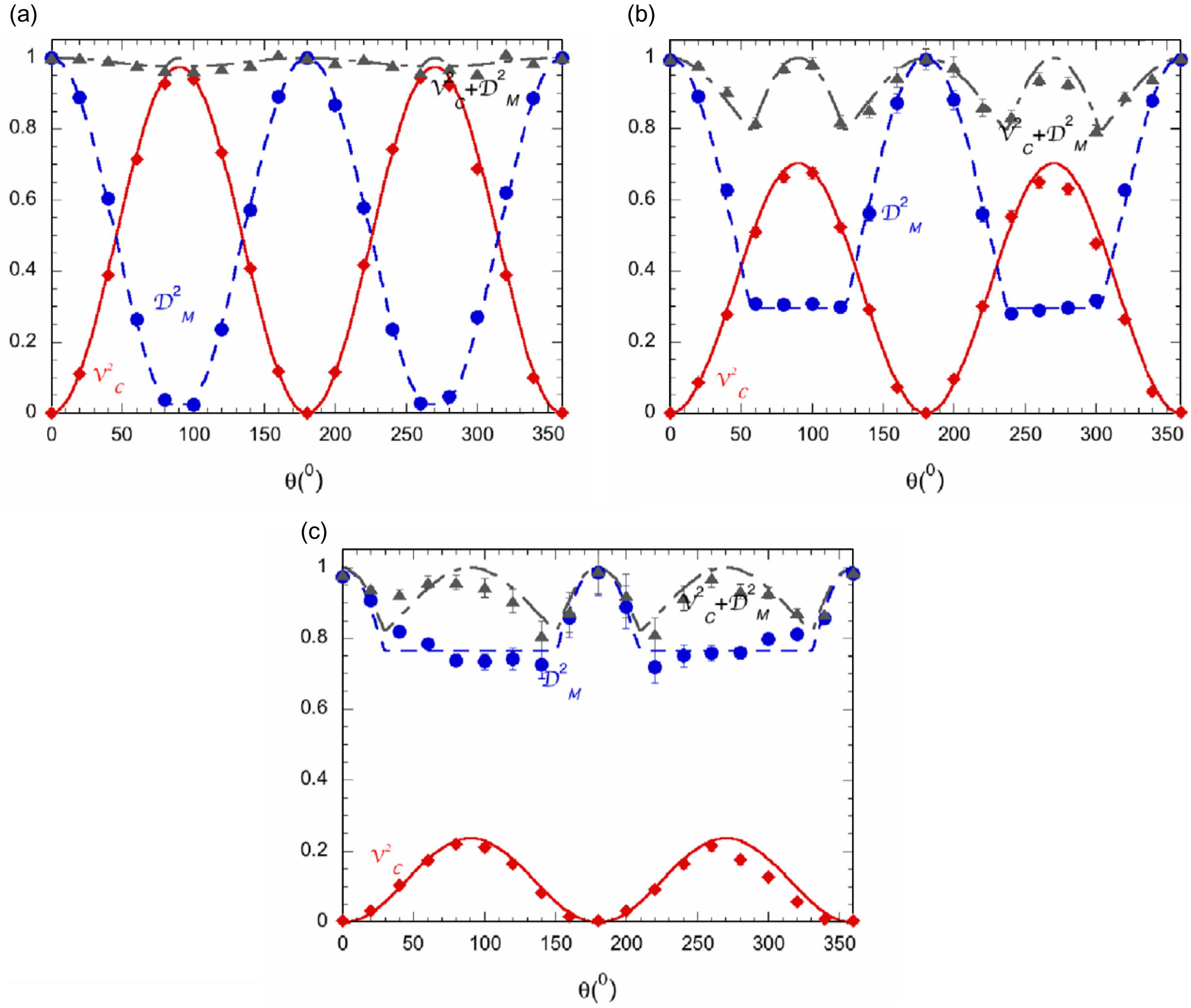

Figure 4. Test of Englert's complementarity relation in the conditional case: conditional visibility $\mathcal{V}_{\mathrm{C}}^2$ (red diamonds), measurement distinguishability $\mathcal{D}_{\mathrm{M}}^2$ (blue circles), and their sum $\mathcal{V}_{\mathrm{C}}^2+\mathcal{D}_{\mathrm{M}}^2$ (grey triangles), for the same values of the *a posteriori* knowledge as above: (a) $\mathcal{K}_{\mathrm{m}}=0.1598\pm0.0091$, (b) $\mathcal{K}_{\mathrm{m}}=0.5445\pm0.0083$ and (c) $\mathcal{K}_{\mathrm{m}}=0.8738\pm0.0058$. The solid lines are the theoretical predictions of equations (3)–(5). When not shown, error bars are smaller than the point size.

which, in our experiment, is

$$\mathcal{D}_{\mathrm{M}}=\max(\mathcal{K}_{\mathrm{s}},\mathcal{K}_{\mathrm{m}}). \tag{4}$$

Similarly, the conditioned visibility $\mathcal{V}_{\mathrm{C}}$ is defined as

$$\mathcal{V}_{\mathrm{C}}=2\left|{}_{\mu}\langle D|_{\sigma}\langle +|\rho_{\sigma\mu}|-\rangle_{\sigma}|D\rangle_{\mu}\right|+2\left|{}_{\mu}\langle A|_{\sigma}\langle +|\rho_{\sigma\mu}|-\rangle_{\sigma}|A\rangle_{\mu}\right|. \tag{5}$$

For these two quantities a generalized Englert's bound holds, $\mathcal{V}_{\mathrm{C}}^2+\mathrm{D}_M^2\leqslant 1$. The results for this measurement are presented in figure 4, and demonstrate the validity of this bound. In our measurements the conditioned visibility and the visibility coincide: this indicates that the extraction of information from the weak measurement is optimal. The data are in good agreement with the somewhat surprisingly discontinuous prediction of (4). This bound is more efficient than Englert's inequality applied to the unsorted data, in the sense that it provides a stricter condition to the visibility including explicitly the role of the QND measurement. Note that the definition of $\mathcal{D}_{\mathrm{M}}$ obtains a complete information balance in the limiting cases when either $\mathcal{K}_{\mathrm{s}} = 0$ or $\mathcal{K}_{\mathrm{m}} = 0$.

There are two complementary explanations for the fact that the optimum is not achieved. Reasoning from an informational point of view, the which-path information is a mixture of *a priori* information from the initial state preparation and *a posteriori* information from the QND. It is possible to extract information from one or the other but not both. $\mathcal{D}_{\mathrm{M}}$ always registers the maximal amount of which-path information available in the system. In our experiment, the extraction of which-path information can be redundant. In fact, if $|\mathcal{K}_{\mathrm{s}}| > |\mathcal{K}_{\mathrm{m}}|$, the QND measurement does not provide any further information with respect to the *a priori* information encoded in the state. The visibility $\mathcal{V}_{\mathrm{C}}$, however, is reduced by the presence of every information channel, even if some are redundant.

On the other hand, the weak measurement correlates the two photon paths to two non-orthogonal polarizations of the meter $|\mu_{\mathrm{in}}\rangle$ and $|\mu_{\mathrm{out}}\rangle$. While this reduces the measurement back-action on the signal, it makes the which-way information partly irretrievable, since it is not possible to discriminate such states with certainty. Therefore, we produced a situation where, although the signal is disturbed and the visibility reduced, the complete information is in principle not attainable.

There is a limiting case in which this approach is effective for an arbitrary transmittivity: when the supplementary channel is erased by an irreversible process. In our experiment, we realize this erasure by projecting the meter photon in the H–V basis. In this case, $\mathcal{D}_{\mathrm{M}} = \mathcal{K}_{\mathrm{s}}$ and $\mathcal{V}_{\mathrm{C}} = \mathcal{V}_{\mathrm{s}}$. Englert's bound is now saturated, since the presence of the additional channel is destroyed, and the visibility is limited only by the original content of which-path knowledge. The data in figures 5(a)–(c) are in very good agreement with the predictions of equations (3)–(5), showing the successful realization of the quantum erasure of the disturbance from the weak measurement [29].

A full analysis of our system takes into account the QND-generated entanglement between the signal and the meter. In this case, a two-particle relation can be written [26, 27]: $\mathcal{V}^2 + \mathcal{D}^2 + \mathcal{C}^2 \leqslant 1$, where $\mathcal{C}$ is the concurrence (i.e. the level of entanglement) of the two-photon state [28]. This limit is saturated for pure states. In our analysis, interpreting $\mathcal{C}$ as an additional source of information would present some difficulties; for instance, it cannot be immediately connected to any measured quantity. We suggest that the expression above expresses a different kind of complementarity: particle, wave or part of an entangled pair.

Quantum information has recently reinterpreted complementarity: when we estimate a quantum state, the classical guess based on measurement limits the fidelity of the post-measurement state with the initial state [30, 31]. Our system performs the optimal strategy for such a protocol; in this case, a bound does not hold for each individual state, but for average quantities. It can be regarded as a complementarity relation applying to the measurement process itself.

In conclusion, we have presented a complete characterization of the complementarity relation including the presence of a weak non-destructive measurement. We find that the visibility is constrained by any which-path knowledge, regardless of its origin.

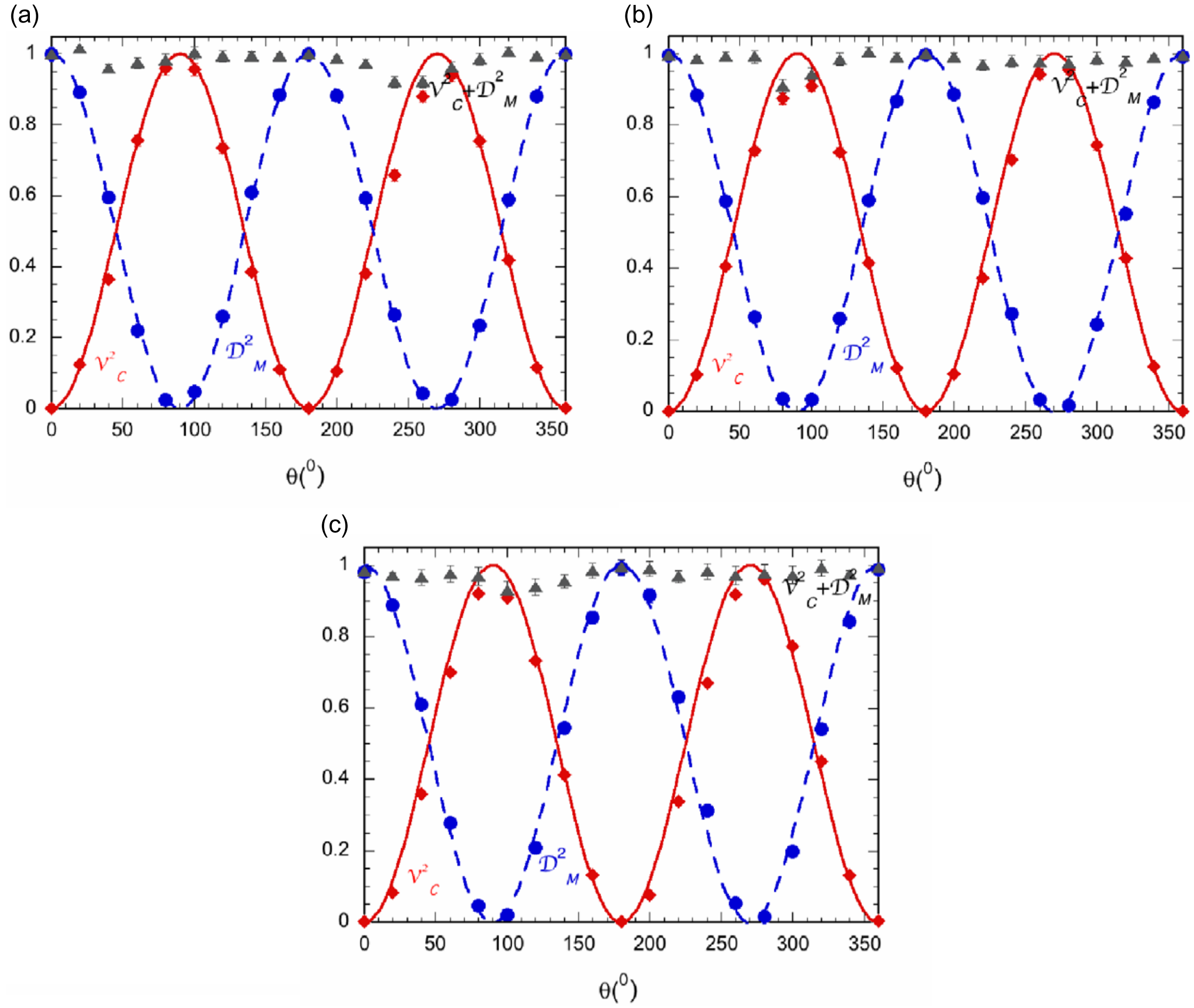

Figure 5. Realisation of the quantum erasure: conditional visibility $\mathcal{V}_{\mathrm{C}}^2$ (red diamonds), measurement distinguishability $\mathcal{D}_{\mathrm{M}}^2$ (blue circles), and their sum $\mathcal{V}_{\mathrm{C}}^2+\mathcal{D}_{\mathrm{M}}^2$ (grey triangles), for the same values of the *a posteriori* knowledge as above: (a) $\mathcal{K}_{\mathrm{m}}=0.1598\pm0.0091$, (b) $\mathcal{K}_{\mathrm{m}}=0.5445\pm0.0083$ and (c) $\mathcal{K}_{\mathrm{m}}=0.8738\pm0.0058$. The solid lines are the theoretical predictions of equations (3)–(5). When not shown, error bars are smaller than the point size.

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